SUCCESSIVE APPROXIMATIONS IN HYDROMECHANICS OF DISPERSE SYSTEMS

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The kinetic equation for particles of the dispersed phase is solved by means of a somewhat modified Chapman-Enskog method used in the kinetic theory of gases. For simplicity only monodisperse systems are considered, in which the effects of direct collisions in the process of momentum and energy exchange between particles is relatively small. Equations of dynamics defining the motion of such systems in mechanics of contiunous media are presented in an approximate form corresponding to the Euler and the Navier-Stokes approximations used in hydromechanics of monophase media.

We are considering here a monodisperse system of particles suspended in a viscous fluid on the assumption that the interaction between particles is primarily due to the random velocity and pressure fields in the fluid, while the effect of particle collisions is negligible. The kinetic equations for such system can be written as [1]

$$\frac{Df}{Dt}+\mathbf{w}'\frac{\partial f}{\partial \mathbf{r}}+\frac{\partial}{\partial \mathbf{w}'}\left[\left(\mathbf{H}^*-\frac{D\langle\mathbf{w}\rangle}{Dt}\right)f\right]-\left(\frac{\partial f}{\partial \mathbf{w}'}*\mathbf{w}'\right)\cdot\cdot\left(\frac{\partial}{\partial \mathbf{r}}*\langle\mathbf{w}\rangle\right)=$$
$$=\left(\frac{\partial}{\partial \mathbf{w}'}*\frac{\partial}{\partial \mathbf{w}'}\right)\cdot\cdot(\mathbf{A}f) \qquad (0.1)$$

$$\mathbf{a}*\mathbf{b}=\|a_ib_j\|,\quad \mathbf{A}\cdot\cdot\mathbf{B}=A_{ij}B_{ji},\qquad \frac{D}{Dt}=\frac{\partial}{\partial t}+\langle\mathbf{w}\rangle\frac{\partial}{\partial \mathbf{r}}$$

Here $f=f(t,\mathbf{r},\mathbf{w}')$ is a unary function of particle distribution with respect to velocity $\mathbf{w}'=\mathbf{w}-\langle\mathbf{w}\rangle$, angle brackets denote averaging over the set, $\mathbf{A}$ is the tensor of diffusion in the velocity space related to the mass of a particle, and force $\mathbf{H}^*$ is defined by the following expression (see Appendix 4):

$$\mathbf{H}^*=\langle\mathbf{H}\rangle+\mathbf{c}\mathbf{w}' \qquad (0.2)$$

$$\langle\mathbf{H}\rangle\approx\mathbf{g}+\varkappa\Big\{\beta_1\Big[K_1\langle\mathbf{u}\rangle+\frac{dK_1}{d\langle\rho\rangle}\langle\rho'\mathbf{u}'\rangle+\frac{1}{2}\frac{d^2K_1}{d\langle\rho\rangle^2}\langle\rho'^2\rangle\langle\mathbf{u}\rangle\Big]+$$
$$+\beta_2\Big[K_2(\langle u\rangle\langle\mathbf{u}\rangle+\langle(\mathbf{u}_0\mathbf{u}')\mathbf{u}'\rangle+{}^1/_2\,\mathbf{u}_0\langle u'^2\rangle-{}^1/_2\langle(\mathbf{u}_0\mathbf{u}')^2\rangle)+$$
$$+\frac{dK_2}{d\langle\rho\rangle}(\langle u\rangle\langle\rho'\mathbf{u}'\rangle+\langle\rho'(\mathbf{u}_0\mathbf{u}')\rangle\langle\mathbf{u}\rangle)+\frac{1}{2}\frac{d^2K_2}{d\langle\rho\rangle^2}\langle\rho'^2\rangle\langle u\rangle\langle\mathbf{u}\rangle\Big]+$$
$$+\frac{D}{Dt}\Big[\xi\langle\mathbf{u}\rangle+\frac{d\xi}{d\langle\rho\rangle}\langle\rho'\mathbf{u}'\rangle+\frac{1}{2}\frac{d^2\xi}{d\langle\rho\rangle^2}\langle\rho'^2\rangle\langle\mathbf{u}\rangle\Big]+$$
$$+\Big\langle\Big(\mathbf{w}'\frac{\partial}{\partial\mathbf{r}}\Big)\Big(\xi\mathbf{u}'+\frac{d\xi}{d\langle\rho\rangle}\rho'\langle\mathbf{u}\rangle\Big)\Big\rangle\Big\}-\frac{1}{d_1}\frac{\partial\langle p\rangle}{\partial\mathbf{r}},\quad \varkappa=\frac{d_0}{d_1}$$

$$K_j\equiv K_j(\langle\rho\rangle),\quad \xi\equiv\xi(\langle\rho\rangle)\qquad (j=1,2)$$

$$\mathbf{c}=\|c_{ij}\|,\quad c_{ij}\approx-\varkappa(\beta_1K_1+\beta_2K_2\langle u\rangle)\delta_{ij}-\varkappa\beta_2K_2\langle u\rangle\delta_{i1}\delta_{j1}+$$
$$+\varkappa\frac{\partial}{\partial r_j}\Big(\xi\langle u_i\rangle\Big),\quad \mathbf{u}=\mathbf{v}-\mathbf{w},\quad \mathbf{u}_0=\frac{\langle\mathbf{u}\rangle}{\langle u\rangle}$$

Here β_j $(j = 1, 2)$ are coefficients dependent on the physical properties of both phases; K_j and ξ are certain functions of $\langle\rho\rangle$ (see Appendix 4); g is the acceleration of the external mass field; v, p and ρ are local values, respectively, of fluid velocity, pressure and volume concentration of the disperse system in the vicinity of a particle moving at velocity w; d_0 and d_1 are the densities of fluid and particle, respectively; the axis $x = r_1$ is directed along the line of the mean velocity of the interphase slip $\langle\mathbf{u}\rangle$.

The momentum equations which follow from (0.1) define the average motion of the dispersed phase as that of a certain continuous medium, and are of the form

$$\frac{D\langle\rho\rangle}{Dt} + \langle\rho\rangle\frac{\partial\langle\mathbf{w}\rangle}{\partial\mathbf{r}} = 0 \qquad (0.3)$$

$$\frac{D\langle\mathbf{w}\rangle}{Dt} = -\frac{1}{\langle\rho\rangle}\frac{\partial(\langle\rho\rangle\,\boldsymbol{\theta})}{\partial\mathbf{r}} + \mathbf{h}^{(p)}, \qquad \mathbf{h}^{(p)} = \langle\mathbf{H}\rangle, \qquad \boldsymbol{\theta} = \langle\mathbf{w}'*\mathbf{w}'\rangle$$

Dynamic equations of the average motion of the fluid phase can be written as [1]

$$\frac{D\langle\rho\rangle}{Dt} = -\langle\mathbf{u}\rangle\frac{\partial\langle\rho\rangle}{\partial\mathbf{r}} + (1-\langle\rho\rangle)\frac{\partial\langle\mathbf{v}\rangle}{\partial\mathbf{r}} + \frac{\partial\mathbf{q}}{\partial\mathbf{r}}, \qquad \mathbf{q} = -\langle\rho'\mathbf{v}'\rangle$$

$$\frac{D\langle\mathbf{v}\rangle}{Dt} = -\left(\langle\mathbf{u}\,\frac{\partial}{\partial\mathbf{r}}\right)\langle\mathbf{v}\rangle - \frac{1}{d_0(1-\langle\rho\rangle)}\frac{\partial\langle p\rangle}{\partial\mathbf{r}} - \mathbf{T}^{(1)} + \frac{2\nu_0}{1-\langle\rho\rangle}\frac{\partial\mathbf{T}^{(2)}}{\partial\mathbf{r}} + \mathbf{h}^{(f)}$$

$$\mathbf{T}^{(1)} = \frac{1}{1-\langle\rho\rangle}\left\{\frac{\partial\mathbf{q}}{\partial t} + \left(\mathbf{q}\,\frac{\partial}{\partial\mathbf{r}}\right)\langle\mathbf{v}\rangle + \frac{\partial}{\partial\mathbf{r}}(\langle\mathbf{v}\rangle*\mathbf{q}) + \frac{\partial}{\partial\mathbf{r}}[(1-\langle\rho\rangle)\,\langle\mathbf{v}'*\mathbf{v}'\rangle]\right\} \qquad (0.4)$$

$$\mathbf{T}^{(2)} = \left(S + \frac{1}{2}\frac{d^2S}{d\langle\rho\rangle^2}\langle\rho'^2\rangle\right)\langle\mathbf{e}_d\rangle + \frac{dS}{d\langle\rho\rangle}\langle\rho'\mathbf{e}_d'\rangle$$

$$\mathbf{h}^{(f)} = \mathbf{g} - \frac{1}{\varkappa}\frac{\langle\rho\rangle}{1-\langle\rho\rangle}(\langle\mathbf{H}\rangle - \mathbf{g}), \qquad S \equiv S(\langle\rho\rangle)$$

where $\mathbf{e}_d$ is the deviator of the deformation rate tensor of fluid flowing at velocity v and function S defines the deviation of apparent viscosity [2] of the fluid filtrating through the grid of particles from its molecular viscosity $\mu_0 = d_0\nu_0$. By definition $\langle\rho\rangle$ and $\langle\mathbf{w}\rangle$ can be expressed in terms of function $f(t, \mathbf{r}, \mathbf{w}')$ as follows:

$$\langle\rho\rangle = \sigma n = \sigma\int f\,d\mathbf{w}', \qquad \langle\mathbf{w}\rangle = \frac{1}{n}\int \mathbf{w}\,f\,d\mathbf{w}' \qquad (0.5)$$

where σ is the particle volume and n the average count of particle concentration.

For the complete determinacy of dynamic equations (0.3), (0.4) all magnitudes of the form $\langle\varphi'\psi'\rangle$ defining the properties of pulsating motions (pseudoturbulence) of the disperse system must be expressed in terms of determinate functions of dynamic variables $\langle\rho\rangle$, $\nabla\langle p\rangle$, $\langle\mathbf{v}\rangle$ and $\langle\mathbf{w}\rangle$ which are the unknowns in these equations. This is the problem considered below.

1. The "equilibrium" distribution function. Let us first consider the equilibrium states of a disperse system in which the dynamic variables are strictly constant. In this case from the dynamic equations (0.3) and (0.4) follow equalities

$$\mathbf{h}_0^{(p)} = 0, \qquad -\frac{1}{d_0(1-\langle\rho\rangle)}\nabla p + \mathbf{h}_0^{(f)} = 0 \qquad (1.1)$$

and the kinetic equation (0.1) becomes

$$\frac{\partial}{\partial\mathbf{w}'}(\mathbf{c}_0\mathbf{w}'f_0) = \left(\frac{\partial}{\partial\mathbf{w}'}*\frac{\partial}{\partial\mathbf{w}'}\right)\cdot\cdot(\mathbf{A}f_0) \qquad (\langle\mathbf{H}\rangle_0 = \mathbf{h}_0^{(p)} = 0) \qquad (1.2)$$

Here and in the following the subscript "zero" denotes magnitudes related to the equilibrium state. The expressions for $\langle \mathbf{H} \rangle_0$ and $\mathbf{c}_0$ are readily derived from (0.2) by neglecting in it terms containing ξ and substituting for all averages of the form $\langle \varphi' \psi' \rangle$ their equilibrium values $\langle \varphi' \psi' \rangle_0$ which can be expressed in terms of dynamic variables by entirely independent means, for example by the method described in [1] (see, also, Appendix 5). It will be readily seen that in the system of coordinates with the axis $x = r_1$ directed along $\mathbf{u}_0$ tensor $\mathbf{c}_0$ is diagonal. With the first of relationships (1.1) taken into account the solution of Eq. (1.2) satisfying the condition of exponentially rapid attenuation when the absolute value of $\mathbf{w}'$ tends to infinity is of the form

$$f_0(\mathbf{w}') = n \left(\frac{1}{8\pi^3 \theta_0^{(1)} \theta_0^{(2)} \theta_0^{(3)}} \right)^{1/2} \exp \left(- \sum_{j=1}^{3} \frac{w_j'^2}{2\theta_0^{(j)}} \right) \tag{1.3}$$

The relationships

$$A^{(j)} = -\theta_0^{(j)} c_0^{(j)}, \qquad \theta_0 = \langle \mathbf{w}' * \mathbf{w}' \rangle_0 \tag{1.4}$$

are then, also, satisfied.

The eigenvalues of tensors $\mathbf{A}$, θ_0 and $\mathbf{c}_0$ in (1.3) and (1.4) are denoted by $A^{(j)}$, $\theta_0^{(j)}$ and $c_0^{(j)}$. The relationships (1.4) completely determine the tensor of diffusion in the velocity space $\mathbf{A}$ appearing in the kinetic equation (0.1).

2. The system of successive approximations. Let the actual state of a disperse system be different from that of equilibrium, so that some of the derivatives of dynamic variables with respect to coordinates and time are not zero. We assume, as in the kinetic theory of gases, that the deviation from equilibrium is small, hence the inequalities

$$L \frac{\partial \ln \langle \varphi \rangle}{\partial r_i} \ll 1 \quad (i = 1, 2, 3), \qquad T \frac{\partial \ln \langle \varphi \rangle}{\partial t} \ll 1 \tag{2.1}$$

where $\langle \varphi \rangle$ is any dynamic variable, and L and T are measures of local pulsating motions (pseudoturbulence) of phases, are satisfied. In this case it is natural to seek the solution of Eq. (0.1) compatible with Eqs. (0.3) and (0.4) in the form of series expansion in a small parameter ε. For this each terms of these equations is to be multiplied by ε^m, where m in the superscript denotes the order of the dynamic variable derivative appearing in such terms (it must be taken into consideration that terms containing zero order derivatives of dynamic variables in (0.3) and (0.4) are, also, of order ε, as can be seen from these equations; an analogous situation arises in the kinetic theory of gases [3]. Parameter ε has no direct physical meaning; as in the Chapman-Enskog method it is introduced here only for the purpose of tracing the order of various terms in expansions of all equations and their solutions. Inclusion of increasingly higher order terms in expansions in ε^m results in a higher accuracy of the definition of the true disequilibrium state of the disperse system. At the end of calculations one must, obviously, assume $\varepsilon = 1$.

Only local-equilibrium states of the disperse system which can be completely defined by dynamic variables at various points and instants of time are considered here. This is tantamount to considering the system in the random-phase approximation used, also, in the kinetic theory of gases (in the latter this approximation is equivalent to the assumption of molecular chaos). In this case function $f(t, \mathbf{r}, \mathbf{w}')$ depends only implicitly on t and $\mathbf{r}$ expressed in terms of dynamic variables, i.e.

$$\frac{Df}{Dt} = \sum_{\varphi} \frac{\partial f}{\partial \langle \varphi \rangle} \frac{D \langle \varphi \rangle}{Dt}, \qquad \frac{\partial f}{\partial \mathbf{r}} = \sum_{\varphi} \frac{\partial f}{\partial \langle \varphi \rangle} \frac{\partial \langle \varphi \rangle}{\partial \mathbf{r}} \tag{2.2}$$

We seek the solution of (0.1) in the form of expansion

$$f = f_0 + \varepsilon f_1 + \varepsilon^2 f_2 + \ldots \tag{2.3}$$

The summations

$$\theta = \sum_{m=0}^{\infty} \varepsilon^m \theta_m, \qquad \theta = \langle \mathbf{w}' * \mathbf{w}' \rangle \tag{2.4}$$

correspond to this expansion. Owing to local equilibrium of the states considered here, the relations between the various pseudoturbulent averages and the components of tensor θ at one and the same point of the disperse system

$$\langle \varphi' \psi' \rangle = R[\varphi, \psi]\,\mathrm{tr}\,\theta, \quad \langle \varphi' \boldsymbol{\psi}' \rangle = \mathbf{R}[\varphi, \boldsymbol{\psi}]\,\theta, \quad \langle \boldsymbol{\varphi}' * \boldsymbol{\psi}' \rangle = \mathbf{R}[\boldsymbol{\varphi}, \boldsymbol{\psi}]\,\theta \tag{2.5}$$

are the same for the equilibrium and the nonequilibrium states [1]. Hence tensors $\mathbf{R}$ introduced in (2.5) can be calculated by these relationships formulated for the equilibrium state from the known characteristics of pseudoturbulence in the equilibrium approximation (see Appendix 5). We note that this property of local-equilibrium states had, in fact, been already used in Sect. 1 for calculating tensor $\mathbf{A}$ appearing in the complete kinetic equation (0.1) by virtue of the solution of the "truncated" equation (1.2), valid only in the true equilibrium state.

Relationships (2.3)–(2.5) make possible the derivation of expansions

$$\langle \varphi' \psi' \rangle = \sum_{m=0}^{\infty} \varepsilon^m \langle \varphi' \psi' \rangle_m, \qquad \langle \varphi' \psi' \rangle_m = R[\varphi, \psi]\,\mathrm{tr}\,\theta_m \tag{2.6}$$

$$\langle \varphi' \boldsymbol{\psi}' \rangle = \sum_{m=0}^{\infty} \varepsilon^m \langle \varphi' \boldsymbol{\psi}' \rangle_m, \qquad \langle \varphi' \boldsymbol{\psi}' \rangle_m = \mathbf{R}[\varphi, \boldsymbol{\psi}]\,\theta_m$$

$$\langle \boldsymbol{\varphi}' * \boldsymbol{\psi}' \rangle = \sum_{m=0}^{\infty} \varepsilon^m \langle \boldsymbol{\varphi}' * \boldsymbol{\psi}' \rangle_m, \qquad \langle \boldsymbol{\psi} * \boldsymbol{\psi}' \rangle_m = \mathbf{R}[\boldsymbol{\varphi}, \boldsymbol{\psi}]\,\theta_m$$

From this and (0.2)–(0.4) we have

$$\mathbf{T}^{(j)} = \sum_{m=0}^{\infty} \varepsilon^m \mathbf{T}_m^{(j)} \qquad (j = 1, 2)$$

$$\mathbf{T}_m^{(1)} = \frac{1}{1 - \langle \rho \rangle} \left\{ \frac{\partial \mathbf{q}_m}{\partial t} + \left(\mathbf{q}_m \frac{\partial}{\partial \mathbf{r}} \right) \langle \mathbf{v} \rangle + \frac{\partial}{\partial \mathbf{r}} (\langle \mathbf{v} \rangle * \mathbf{q}_m) + \frac{\partial}{\partial \mathbf{r}} [(1 - \langle \rho \rangle) \langle \mathbf{v}' * \mathbf{v}' \rangle_m] \right\}$$

$$\mathbf{T}_m^{(2)} = \left(S \delta_{0m} + \frac{1}{2} \frac{d^2 S}{d \langle \rho \rangle^2} \langle \rho'^2 \rangle_m \right) \langle \mathbf{e}_d \rangle + \frac{dS}{d \langle \rho \rangle} \langle \rho' \mathbf{e}_d' \rangle_m$$

$$\mathbf{h}^{(p)} = \sum_{m=0}^{\infty} \varepsilon^m \mathbf{h}_m^{(p)}, \qquad \mathbf{h}^{(f)} = \sum_{m=0}^{\infty} \varepsilon^m \mathbf{h}_m^{(f)}$$

$$\mathbf{h}_m^{(p)} = \left[\mathbf{g} + \varkappa (\beta_1 K_1 \langle \mathbf{u} \rangle + \beta_2 K_2 \langle u \rangle \langle \mathbf{u} \rangle) + \varkappa \frac{D}{Dt} (\xi \langle \mathbf{u} \rangle) - \frac{1}{d_1} \frac{\partial \langle p \rangle}{\partial \mathbf{r}} \right] \delta_{0m} +$$
$$+ \varkappa \left\{ \beta_1 \left(\frac{dK_1}{d \langle \rho \rangle} \langle \rho' \mathbf{u}' \rangle_m + \frac{1}{2} \frac{d^2 K_1}{d \langle \rho \rangle^2} \langle \rho'^2 \rangle_m \langle \mathbf{u} \rangle \right) + \beta_2 [K_2 (\langle (\mathbf{u}_0 \mathbf{u}') \mathbf{u}' \rangle_m + \right.$$
$$+ \frac{1}{2} \mathbf{u}_0 \langle u'^2 - (\mathbf{u}_0 \mathbf{u}')^2 \rangle_m) + \frac{dK_2}{d \langle \rho \rangle} (\langle u \rangle \langle \rho' \mathbf{u}' \rangle_m + \langle \rho' (\mathbf{u}_0 \mathbf{u}') \rangle_m \langle \mathbf{u} \rangle) +$$
$$+ \frac{1}{2} \frac{d^2 K_2}{d \langle \rho \rangle^2} \langle \rho'^2 \rangle_m \langle u \rangle \langle \mathbf{u} \rangle] + \frac{D}{Dt} \left(\frac{d\xi}{d \langle \rho \rangle} \langle \rho' \mathbf{u}' \rangle_m + \frac{1}{2} \frac{d^2 \xi}{d \langle \rho \rangle^2} \langle \rho'^2 \rangle_m \langle \mathbf{u} \rangle \right) +$$
$$\left. + \left\langle \left(\mathbf{w}' \frac{\partial}{\partial \mathbf{r}} \right) \left(\xi \mathbf{u}' + \frac{d\xi}{d \langle \rho \rangle} \rho' \langle \mathbf{u} \rangle \right) \right\rangle_m \right\} \tag{2.7}$$

$$\mathbf{h}_m^{(f)} = \left(1 - \frac{1}{\varkappa}\frac{\langle\rho\rangle}{1-\langle\rho\rangle}\right)\mathbf{g}\delta_{0m} - \frac{1}{\varkappa}\frac{\langle\rho\rangle}{1-\langle\rho\rangle}\mathbf{h}_m^{(p)} \qquad \text{(cont.)}$$

To obtain a self-consistent scheme for successive approximations it is necessary to define functions f_m so that, by neglecting all terms of expansion (2.3) of order higher than a certain number M and, also, all related terms in expansions (2.4), (2.6), and (2.7), the Mth approximation for the distribution function and all dynamic variables is actually obtained. The significance of this self-consistency requirement is the same as in the case of solution of Boltzmann's kinetic equation by the Chapman-Enskog method [3].

Let us write the convection derivatives of f and of the dynamic variable $\langle\varphi\rangle$ in the following form:

$$\frac{Df}{Dt} = \sum_{m=0}^{\infty} \varepsilon^m \frac{D_m f}{Dt}, \qquad \frac{D\langle\varphi\rangle}{Dt} = \sum_{m=0}^{\infty}\varepsilon^m \frac{D_m\langle\varphi\rangle}{Dt} \tag{2.8}$$

These expansions are to be defined so as to conform with the laws of mass and momentum conservation defined by Eqs. (0.3) and (0.4). It will be readily seen that this implies the following definition of the various terms in (2.8):

$$\frac{D_m\langle\rho\rangle}{Dt} = \left[-\langle\mathbf{u}\rangle\frac{\partial\langle\rho\rangle}{\partial\mathbf{r}} + (1-\langle\rho\rangle)\frac{\partial\langle\mathbf{v}\rangle}{\partial\mathbf{r}}\right]\delta_{m0} + \frac{\partial\mathbf{q}_m}{\partial\mathbf{r}}$$

$$\frac{D_m\langle\mathbf{w}\rangle}{Dt} = -\frac{1}{\langle\rho\rangle}\frac{\partial(\langle\rho\rangle\,\theta_m)}{\partial\mathbf{r}} + \mathbf{h}_m^{(p)} \tag{2.9}$$

$$\frac{D_m\langle\mathbf{v}\rangle}{Dt} = -\left[\left(\langle\mathbf{u}\rangle\frac{\partial}{\partial\mathbf{r}}\right)\langle\mathbf{v}\rangle + \frac{1}{d_0(1-\langle\rho\rangle)}\frac{\partial\langle p\rangle}{\partial\mathbf{r}}\right]\delta_{m0} - \mathbf{T}_m^{(1)} + \frac{2\nu_0}{1-\langle\rho\rangle}\frac{\partial\mathbf{T}_{m-1}^{(2)}}{\partial\mathbf{r}} + \mathbf{h}_m^{(f)}$$

Moreover, from the first of Eqs. (0.3) follows

$$\frac{\partial\langle\mathbf{w}\rangle}{\partial\mathbf{r}} = \sum_{m=0}^{\infty}\varepsilon^m \operatorname{div}_m\langle\mathbf{w}\rangle, \qquad \operatorname{div}_m\langle\mathbf{w}\rangle = -\frac{D_m \ln\langle\rho\rangle}{Dt} \tag{2.10}$$

The convection derivative of pressure can be determined from the trivial equalities

$$\frac{D_0\langle p\rangle}{Dt} \equiv \frac{D\langle p\rangle}{Dt}, \qquad \frac{D_m\langle p\rangle}{Dt} \equiv 0 \quad (m>0) \tag{2.11}$$

With the use of (2.2) and (2.3) for the convection derivative of f we obtain

$$\frac{Df}{Dt} = \sum_{\varphi}\left(\sum_{m'=0}^{\infty}\varepsilon^{m'}\frac{D_{m'}\langle\varphi\rangle}{Dt}\right)\left(\sum_{m''=0}^{\infty}\varepsilon^{m''}\frac{\partial f_{m''}}{\partial\langle\varphi\rangle}\right) \tag{2.12}$$

From (2.8) and (2.12) we also have

$$\frac{D_0 f}{Dt} = \sum_{\varphi}\frac{\partial f_0}{\partial\langle\varphi\rangle}\frac{D_0\langle\varphi\rangle}{Dt}, \qquad \frac{D_1 f}{Dt} = \sum_{\varphi}\left(\frac{\partial f_0}{\partial\langle\varphi\rangle}\frac{D_1\langle\varphi\rangle}{Dt} + \frac{\partial f_1}{\partial\langle\varphi\rangle}\frac{D_0\langle\varphi\rangle}{Dt}\right) \tag{2.13}$$

It is expedient to chose for f_0 the equilibrium form (1.3). Then

$$\frac{D_0 f_0}{Dt} = \frac{\partial f_0}{\partial\langle\rho\rangle}\frac{D_0\langle\rho\rangle}{Dt} + \sum_{j=1}^{3}\frac{\partial f_0}{\partial\theta_0^{(j)}}\sum_{\varphi}\frac{\partial\theta_0^{(j)}}{\partial\langle\varphi\rangle}\frac{D_0\langle\varphi\rangle}{Dt} \tag{2.14}$$

where, in accordance with the foregoing, $\theta_0^{(j)}$ are considered to be known functions of dynamic variables $\langle\varphi\rangle$.

Introducing parameter ε into Eqs. (0.1) and (0.3) and separating in these terms of different order with respect to ε, for the various coefficients f_m in expansion (2.3) we obtain

$$\sum_{j=1}^{3} A^{(j)}\left[\frac{\partial^2 f_m}{\partial w_j'^2}+\frac{1}{\theta_0^{(j)}}\frac{\partial (w_j' f_m)}{\partial w_j'}\right]=\frac{D_{m-1}f}{Dt}+\mathbf{w}'\frac{\partial f_{m-1}}{\partial \mathbf{r}}+ \tag{2.15}$$

$$+\frac{\partial}{\partial \mathbf{w}'}(\mathbf{c}_1\mathbf{w}' f_{m-1})+\frac{1}{\langle\rho\rangle}\sum_{m'=0}^{m-1}\frac{\partial(\langle\rho\rangle\theta_{m'})}{\partial \mathbf{r}}\frac{\partial f_{m-1-m'}}{\partial \mathbf{w}'}-\left(\frac{\partial f_{m-1}}{\partial \mathbf{r}}*\mathbf{w}'\right)\cdot\cdot\left(\frac{\partial}{\partial \mathbf{r}}*\langle\mathbf{w}\rangle\right)$$

Relationships (1.4) were also used in the formulation of (2.15). The expression for the tensor $\mathbf{c}_1$ is of the form (see (0.2))

$$\mathbf{c}_1=\|c_{1,ij}\|,\qquad c_{1,ij}=\varkappa\frac{\partial}{\partial r_j}(\xi\langle u_i\rangle) \tag{2.16}$$

Solutions of Eqs. (2.15) must satisfy conditions

$$\int f_m\,d\mathbf{w}'=0,\quad \int \mathbf{w}' f_m\,d\mathbf{w}'=0,\quad m>0 \tag{2.17}$$

The dynamic equations corresponding to the mth approximation are

$$\frac{D\langle\rho\rangle}{Dt}+\langle\rho\rangle\frac{\partial\langle\mathbf{w}\rangle}{\partial \mathbf{r}}=0$$

$$d_1\langle\rho\rangle\frac{D\langle\mathbf{w}\rangle}{Dt}=-d_1\frac{\partial}{\partial \mathbf{r}}\left(\langle\rho\rangle\sum_{m'=0}^{m}\theta_{m'}\right)+d_1\langle\rho\rangle\sum_{m'=0}^{m}\mathbf{h}_{m'}^{(p)}$$

$$\frac{D_v\langle\rho\rangle}{Dt}-(1-\langle\rho\rangle)\frac{\partial\langle\mathbf{v}\rangle}{\partial \mathbf{r}}-\frac{\partial}{\partial \mathbf{r}}\sum_{m'=0}^{m}\mathbf{q}_{m'}=0 \tag{2.18}$$

$$d_0(1-\langle\rho\rangle)\frac{D_v\langle\mathbf{v}\rangle}{Dt}=-\frac{\partial\langle p\rangle}{\partial \mathbf{r}}-d_0(1-\langle\rho\rangle)\sum_{m'=0}^{m}\mathbf{T}_{m'}^{(1)}+2\mu_0\frac{\partial}{\partial \mathbf{r}}\sum_{m'=0}^{m-1}\mathbf{T}_{m'}^{(2)}+$$

$$+d_0(1-\langle\rho\rangle)\sum_{m'=0}^{m}\mathbf{h}_{m'}^{(f)},\quad \frac{D_v}{Dt}=\frac{\partial}{\partial t}+\langle\mathbf{v}\rangle\frac{\partial}{\partial \mathbf{r}}$$

Parameters θ_m appearing here are calculated in the usual manner from the determined beforehand magnitudes f_m

$$\theta_m=\frac{1}{n}\int(\mathbf{w}'*\mathbf{w}')f_m\,d\mathbf{w}' \tag{2.19}$$

The various pseudoturbulent averages are expressed in terms of θ_m and of dynamic variables with the use of relationships (2.5), while $\mathbf{T}_m^{(1)}$, $\mathbf{T}_m^{(2)}$, $\mathbf{h}_m^{(p)}$, $\mathbf{h}_m^{(f)}$, and $\mathbf{q}_m$ are calculated in conformity with their definitions in (2.7).

It is possible in principle, to determine the dynamic equations (2.18) with any desired accuracy by successively solving Eqs. (2.15). It would appear, however, that for the most practical applications the analysis can be limited to the first two approximations ($m = 1, 2$) which, by analogy to hydromechanics of monophase media and the kinetic theory, may be appropriately called the Euler and the Navier-Stokes approximations, respectively. There is here an obvious analogy to gasdynamics, where it is generally sufficient in investigations of macroscopic motions of gas to use equations of hydrodynamics in the form of Euler or Navier-Stokes approximations, and the need to resort to equations of a higher order of approximation (e.g. the Barnett equations) does not arise.

We would point out the important difference between the method used here and the conventional Chapman-Enskog method in the kinetic theory. According to the latter the isotropic magnitude $\theta = \operatorname{tr} \boldsymbol{\theta}$ defining the temperature of gas is considered, along with the gas density and the mean velocity of its motion, to be an independent parameter. In the present analysis the mean squares of particle velocity components pulsating in various directions are, on the contrary, entirely determinate functions of dynamic variables and, in particular, of $\langle\rho\rangle$ and $\langle\mathbf{w}\rangle$. From the physical point of view this difference is entirely reasonable.

In fact, unlike an ordinary gas which is a two-parameter system (whose equilibrium state is completely defined by only two independent parameters, e.g. density and temperature), the suspended particles represent a single-parameter system. This will be readily seen, if one considers that the state of a disperse system is entirely defined by specifying a single parameter, such as the mean concentration of particles or the mean velocity of the interphase slip.

The situation in which the energy of pulsating motions is independent of properties of the flow of the disperse system can, in principle, occur only if some additional sources of pseudoturbulent motion of particles were introduced. It seems that in practice this kind of situation can only exist in the vicinity of the flow boundaries. Solid boundaries, obviously, contribute to the attenuation of pseudoturbulence in their vicinity, independently of the physical properties of particles and of the dispersing medium. Coarse disperse grids permeable only to the dispersing medium can, on the other hand, induce pseudoturbulence by distorting the flow of the dispersing medium and the subsequent interaction between the perturbed stream (i.e. the small streams flowing through individual openings in the grid) with particles. An example of a boundary of the second kind is provided by the grid admitting the fluidizing medium to the fluidized bed from below.

The foregoing shows that it is not necessary to take into account the transport equations for the tensor $\boldsymbol{\theta}$ component, when solving the kinetic equations for suspended particles. Such equations are, however, readily derived from (0.1) by conventional methods. For example the transport equation for the first invariant of tensor $\boldsymbol{\theta}$ is of the form

$$\frac{D\theta}{Dt} + 2\boldsymbol{\theta}\cdot\cdot\left(\frac{\partial}{\partial\mathbf{r}} * \langle\mathbf{w}\rangle\right) - 2\boldsymbol{\theta}\cdot\cdot\mathbf{c} + 2\operatorname{tr}\mathbf{A} + \frac{1}{\langle\rho\rangle}\frac{\partial\mathbf{Q}}{\partial\mathbf{r}} = 0 \qquad (2.20)$$

$$\theta = \operatorname{tr}\boldsymbol{\theta}, \quad \mathbf{Q} = \sigma\int w'^2\mathbf{w}' f d\mathbf{w}', \quad \mathbf{c} = \mathbf{c}_0 + \mathbf{c}_1$$

This equation may be considered as the equation of pseudoturbulent energy transport of suspended particles (the mean energy of a single particle pulsation is obviously equal to $^1/_2\, m\theta$, where m is the mass of a particle); it is, thus, analogous to the thermal conductivity equation in the kinetic theory or in hydromechanics of monophase media. The various terms in the left-hand side of (2.20) define, respectively, the convective variation of θ, the intensification of pseudoturbulence owing to dissipation of energy of the dispersed phase mean motion by pseudoturbulent stresses appearing in the second of Eqs. (2.18), the normal degeneration of pulsating motions produced by forces of interaction between the phases, the "Brownian" generation of pulsations by random small scale perturbations, and, finally, the transport of θ by the particle pseudoturbulent pulsations. We note that it is not difficult to derive equations for $\theta_m = \operatorname{tr}\boldsymbol{\theta}_m$, corresponding to Eq. (2.20), in which $\boldsymbol{\theta}_m$ are determined by (2.19).

Thus, the here described method differs somewhat from that of Chapman-Enskog in which θ, proportional to the gas temperature, is considered to be an independent parameter, and its equations are used for solving the Boltzmann kinetic equations together with those of mass and momentum conservation [3].

3. The Euler and Navier-Stokes approximations. Let us select as the first term of expansion (2.3) the function f_0 defined by (1.3). The related dynamic equations are derived from (2.7) and (2.18) for $m=0$, i.e. after substitution for the actual characteristics of pseudoturbulence of the equilibrium values of these. As noted earlier, the latter are determined independently (see [1] and Appendix 5 below) and can be considered as known functions of the dynamic variables. This makes it possible to close the system of dynamic equations defining the average motion of the disperse system phases in the "zero" approximation considered here. In hydromechanics of disperse systems the latter may be appropriately called the Euler approximation.

In the system of coordinates in which the axis $x = r_1$ is directed along $\mathbf{u}_0$, the relationship $\mathbf{q}_0 = \{q_0, 0, 0\}$ is entirely satisfied and the tensors θ_0 and $\mathbf{T}_0^{(1)}$ are diagonal. Hence in the Euler approximation only the normal pseudoturbulent stresses in both phases, and the component of the fluid phase pulsating stream in the direction of the interphase slip mean velocity are actually taken into consideration. The normal pseudoturbulent stresses are analogous to pressure in the kinetic theory of gases, while the pulsating stream is similar to an additional stream of compressible fluid produced by turbulent pulsations of its density and velocity. The transport equation for $\dot{\theta}_0$ is of the form (2.20), but the flux $\mathbf{Q}_0$ is obviously identically zero.

Let us now consider the following approximation with respect to parameter ε, i.e. with respect to the deviation of the true state of the disperse system from that of equilibrium. We introduce the new unknown function g_1:

$$f_1 = f_0 g_1 \tag{3.1}$$

For $m = 1$ Eq. (2.15) can be rewritten in the form

$$\sum_{j=1}^{3} A^{(j)} \left(\frac{\partial^2}{\partial w_j'^2} - \frac{w_j'}{\theta_0^{(j)}} \frac{\partial}{\partial w_j'} \right) g_1 = \frac{1}{f_0} \left[\frac{D_0 f}{Dt} + \mathbf{w}' \frac{\partial f_0}{\partial \mathbf{r}} + \frac{\partial}{\partial \mathbf{w}'} (\mathbf{c}_1 \mathbf{w}' f_0) + \right.$$
$$+ \frac{1}{\langle \rho \rangle} \frac{\partial (\langle \rho \rangle \theta_0)}{\partial \mathbf{r}} \frac{\partial f_0}{\partial \mathbf{w}'} - \left(\frac{\partial f_0}{\partial \mathbf{w}'} * \mathbf{w}' \right) \cdot\cdot \left(\frac{\partial}{\partial \mathbf{r}} * \langle \mathbf{w} \rangle - \mathbf{C} \right) - \tag{3.2}$$
$$\left. - \left(\frac{\partial f_0}{\partial \mathbf{w}'} * \mathbf{w}' \right) \cdot\cdot \mathbf{C} \right], \quad \mathbf{C} = \| C_{ij} \|, \quad C_{ij} = \frac{\partial \langle w_i \rangle}{\partial r_i} \delta_{ij}$$

Using relationships (1.3) and (2.9)–(2.14) for transforming the right-hand of this equation, we obtain the following expressions:

$$\frac{1}{f_0} \frac{D_0 f}{Dt} = -\operatorname{div}_0 \langle \mathbf{w} \rangle + \frac{1}{2} \sum_{j=1}^{3} \left(\frac{w_j'^2}{\theta_0^{(j)}} - 1 \right) \frac{D_0 \ln \theta_0^{(j)}}{Dt}$$
$$\frac{\mathbf{w}'}{f_0} \frac{\partial f_0}{\partial \mathbf{r}} = \mathbf{w}' \frac{\partial \ln \langle \rho \rangle}{\partial \mathbf{r}} + \frac{1}{2} \sum_{j=1}^{3} \left(\frac{w_j'^2}{\theta_0^{(j)}} - 1 \right) \mathbf{w}' \frac{\partial \ln \theta_0^{(j)}}{\partial r}$$
$$\frac{1}{f_0 \langle \rho \rangle} \frac{\partial (\langle \rho \rangle \theta_0)}{\partial \mathbf{r}} \frac{\partial f_0}{\partial \mathbf{w}'} = -\mathbf{w}' \frac{\partial \ln \langle \rho \rangle}{\partial \mathbf{r}} - \sum_{j=1}^{3} w_j' \frac{\partial \ln \theta_0^{(j)}}{\partial r_j} \tag{3.3}$$
$$\frac{1}{f_0} \frac{\partial}{\partial \mathbf{w}'} (\mathbf{c}_1 \mathbf{w}' f_0) = -\sum_{j=1}^{3} c_1^{(j)} \left(\frac{w_j'^2}{\theta_0^{(j)}} - 1 \right) - \sum_{i,j=1}^{3} \Gamma_{ij}' w_i' w_j'$$

$$-\frac{1}{f_0}\left(\frac{\partial f_0}{\partial \mathbf{w}'}*\mathbf{w}'\right)\cdot\cdot\left(\frac{\partial}{\partial \mathbf{r}}*\langle\mathbf{w}\rangle-\mathbf{C}\right)=\sum_{i,j=1}^{3}\Gamma_{ij}''w_i'w_j' \qquad \text{(cont.)}$$

$$-\frac{1}{f_0}\left(\frac{\partial f_0}{\partial \mathbf{w}'}*\mathbf{w}'\right)\cdot\cdot\mathbf{C}=\sum_{j=1}^{3}\frac{w_j'^2}{\theta_0^{(j)}}\frac{\partial\langle w_j\rangle}{\partial r_j}$$

We have introduced here symmetric tensors Γ' and Γ''

$$\Gamma'=\|\Gamma_{ij}'\|,\qquad \Gamma''=\|\Gamma''_{ij}\|$$

$$\Gamma_{ij}'=-\frac{1}{2}\left(\frac{c_{1,ij}}{\theta_0^{(j)}}+\frac{c_{1,ji}}{\theta_0^{(i)}}\right)+\frac{c_1^{(j)}}{\theta_0^{(j)}}\delta_{ij} \tag{3.4}$$

$$\Gamma_{ij}''=\frac{1}{2}\left(\frac{1}{\theta_0^{(j)}}\frac{\partial\langle w_j\rangle}{\partial r_i}+\frac{1}{\theta_0^{(i)}}\frac{\partial\langle w_i\rangle}{\partial r_j}\right)-\frac{1}{\theta_0^{(j)}}\frac{\partial\langle w_j\rangle}{\partial r_j}\delta_{ij}$$

Substituting relationships (3.3) and (3.4) into Eq. 3.2), we obtain

$$\sum_{j=1}^{3}A^{(j)}\left(\frac{\partial^2}{\partial w_j^2}-\frac{w_j'}{\theta_0^{(j)}}\frac{\partial}{\partial w_j'}\right)g_1=\sum_{j=1}^{3}\left(\frac{w_j'^2}{\theta_0^{(j)}}-1\right)\left(\frac{D_0\ln\theta_0^{(j)}}{2Dt}-c_1^{(j)}+\frac{\partial\langle w_j\rangle}{\partial r_j}\right)+$$
$$+\sum_{i,j=1}^{3}\Gamma_{ij}w_i'w_j'+\frac{1}{2}\sum_{i,j=1}^{3}\left(\frac{w_j'^2}{\theta_0^{(j)}}-1\right)\mathbf{w}'\frac{\partial\ln\theta_0^{(j)}}{\partial\mathbf{r}} \tag{3.5}$$

We have here tensor Γ (see (2.16) and (3.4))

$$\Gamma=\|\Gamma_{ij}\|=\Gamma''-\Gamma',\qquad W=\langle\mathbf{w}\rangle+\varkappa\xi\langle\mathbf{u}\rangle \tag{3.6}$$

$$\Gamma_{ij}=\frac{1}{2}\left(\frac{1}{\theta_0^{(j)}}\frac{\partial W_j}{\partial r_i}+\frac{1}{\theta_0^{(i)}}\frac{\partial W_i}{\partial r_j}\right)-\frac{1}{\theta_0^{(i)}}\frac{\partial W_j}{\partial r_j}\delta_{ij}$$

We seek the solution of Eq. (3.5) in the form

$$g_1=K+\sum_{j=1}^{3}L_jw_j'+\sum_{i,j=1}^{3}(M_{ij}+N_{ij}w_j')w_i'w_j' \tag{3.7}$$

where K, L_j, M_{ij}, N_{ij} are certain quantities independent of components $\mathbf{w}'$. Substituting (3.7) into Eq. (3.5) and equating terms of the same powers of w_i' in various parts of this equation, we obtain for these quantities

$$N_{jj}=-\frac{1}{6A^{(j)}}\frac{\partial\ln\theta_0^{(j)}}{\partial r_j},\quad N_{ij}=-\frac{1}{2}\left(\frac{2A^{(j)}}{\theta_0^{(j)}}+\frac{A^{(i)}}{\theta_0^{(i)}}\right)^{-1}\frac{1}{\theta_0^{(j)}}\frac{\partial\ln\theta_0^{(j)}}{\partial r_i},\quad i\neq j$$

$$M_{jj}=-\frac{1}{2A^{(j)}}\left(\frac{D_0\ln\theta_0^{(j)}}{2Dt}-c_1^{(j)}+\frac{\partial\langle w_j\rangle}{\partial r_j}\right)$$

$$M_{ij}=-\left(\frac{A^{(j)}}{\theta_0^{(j)}}+\frac{A^{(i)}}{\theta_0^{(i)}}\right)^{-1}\Gamma_{ij},\quad i\neq j \tag{3.8}$$

$$L_j=\frac{\theta_0^{(j)}}{A^{(j)}}\left(6A^{(j)}N_{jj}+2\sum_{i=1,\,i\neq j}^{3}N_{ji}A^{(i)}+\frac{1}{2}\sum_{i=1,\,i\neq j}^{3}\frac{\partial\ln\theta_0^{(i)}}{\partial r_j}+\frac{3}{2}\frac{\partial\ln\theta^{(j)}}{\partial r_j}\right)=$$
$$=\frac{1}{2}\sum_{i=1,\,i\neq j}^{3}\left(\frac{2A^{(i)}}{\theta_0^{(i)}}+\frac{A^{(j)}}{\theta_0^{(j)}}\right)^{-1}\frac{\partial\ln\theta_0^{(i)}}{\partial r_j}+\frac{1}{2}\cdot\frac{\theta_0^{(j)}}{A^{(j)}}\frac{\partial\ln\theta_0^{(j)}}{\partial r_j}$$

Let us check whether conditions (2.17) are satisfied. We have

$$\int f_1 d\mathbf{w}' = \int g_1 f_0 d\mathbf{w}' = \int \Big(K + \sum_{j=1}^{3} M_{jj} w_j'^2\Big) f_0 d\mathbf{w}' =$$
$$= n\Big[K - \frac{1}{2}\sum_{j=1}^{3} \frac{\theta_0^{(j)}}{A^{(j)}}\Big(\frac{D_0 \ln \theta_0^{(j)}}{2Dt} - c_1^{(j)} + \frac{\partial \langle w_j\rangle}{\partial r_j}\Big)\Big] \tag{3.9}$$

Equating expression (3.9) to zero, for K appearing in relationship (3.7) we obtain

$$K = \frac{1}{2}\sum_{j=1}^{3} \frac{\theta_0^{(j)}}{A^{(j)}}\Big(\frac{D_0 \ln \theta_0^{(j)}}{2Dt} - c_1^{(j)} + \frac{\partial \langle w_j\rangle}{\partial r_j}\Big) \tag{3.10}$$

The quantities appearing in parentheses in (3.8) and (3.10) can be readily expressed in the form of determinate functions of dynamic variables. For this it is sufficient to express the convection derivatives of $\theta_0^{(j)}$ by the equalities

$$\frac{D_0 \ln \theta_0^{(j)}}{Dt} = \frac{\partial \ln \theta_0^{(j)}}{\partial \langle \rho \rangle}\frac{D_0\langle \rho\rangle}{Dt} + \frac{\partial \ln \theta_0^{(j)}}{\partial \langle u\rangle}\frac{D_0 \langle u\rangle}{Dt} \tag{3.11}$$

(as shown in [1] and Appendix 5, $\theta_0^{(j)}$ are independent of $\langle p\rangle$) and, then, make use of relationship (2.9) and of the known expressions for the characteristics of equilibrium pseudoturbulence $\langle \varphi'\psi'\rangle_0$ in terms of dynamic variables. The second of conditions (2.17) is identically satisfied. In fact, we have

$$\int w_j' f_1 d\mathbf{w}' = \int w_j' g_1 f_0 d\mathbf{w}' = \int w_j'\Big(L_j w_j' + \sum_{i=1}^{3} N_{ji} w_j' w_i'^2\Big) f_0 d\mathbf{w}' =$$
$$= \frac{1}{2}\sum_{i=1, i\neq j}^{3}\Big(\frac{2A^{(i)}}{\theta_0^{(i)}} + \frac{A^{(j)}}{\theta_0^{(j)}}\Big)^{-1}\frac{\partial \ln \theta_0^{(i)}}{\partial r_j}\int w_j'^2\Big(1 - \frac{w_i'^2}{\theta_0^{(i)}}\Big) f_0 dw' + \tag{3.12}$$
$$+ \frac{1}{2}\frac{\theta_0^{(j)}}{A^{(j)}}\frac{\partial \ln \theta_0^{(j)}}{\partial r_j}\int w_j'^2\Big(1 - \frac{w_j'^2}{3\theta_0^{(j)}}\Big) d\mathbf{w}' \equiv 0$$

Thus the solution (3.1), (3.7) of Eq. (2.15) satisfies for $m = 1$ both conditions (2.17).

The components of tensor θ_1 are

$$\theta_{1,ij} = \frac{1}{n}\int w_i' w_j' f_1 d\mathbf{w}' = \frac{1}{n}\int w_i' w_j' g_1 f_0 d\mathbf{w}' =$$
$$= \sum_{k=1}^{3}(1 + \delta_{ik})\frac{\theta_0^{(k)}}{A^{(k)}}\Big(\frac{D_0 \ln \theta_0^{(k)}}{2Dt} - c_1^{(k)} + \frac{\partial \langle w_k\rangle}{\partial x_j}\Big)\theta_0^{(i)}\delta_{ij} -$$
$$- \theta_0^{(i)}\theta_0^{(j)}\Big(\frac{A^{(i)}}{\theta_0^{(i)}} + \frac{A^{(j)}}{\theta_0^{(j)}}\Big)^{-1}\Gamma_{ij} \tag{3.13}$$

The dynamic equations for $m = 1$ are derived from Eqs. (2.18) by substituting into these relationships (2.6), (2.7), (3.6), and (3.13). We note the appearance of new terms in the dynamic equations in this approximation (which in the hydromechanics of disperse systems may be appropriately called the Navier-Stokes approximations) which differ from the corresponding equations in the Euler approximation.

First of all, along with the component q_1 of the pulsating stream $\mathbf{q}$ there also appear transverse components normal to vector $\mathbf{u}_0$. Next, the normal pseudoturbulent stresses present in both phases, considered to be continuous media, are somewhat changed and, what is particularly important, there appear tangential pseudoturbulent stresses similar in their meaning to viscous stresses in a monophase medium or to stresses induced by turbulent viscosity. Finally, the forces $\mathbf{h}^{(p)}$ and $\mathbf{h}^{(f)}$, appearing in the equations of momentum conservation of phases in (2.18), change somewhat.

The expressions for tangential pseudoturbulent stresses differ substantially from those specified in the various phenomenological models of disperse systems. In particular, it is generally difficult to separate in the tensors of pseudoturbulent stresses of both phases the viscosity and the mean deformation rate tensors; it will be readily seen that the introduction of scalar coefficients of turbulent viscosity is possible only in the case of the simplest of flows (e.g. one-dimensional flow), and the values of these depend to a great extent on the kind, orientation, etc., of the flow.

It will be readily seen that in the transport equations derived from (2.4) and (2.20) the stream $Q = Q_1$ is not zero. The expression defining this stream is easily obtained by using (2.19) and (3.1) as input equations.

Equations (2.18) for $m = 0$ or $m = 1$ prove to be considerably more complicated than the usual Euler or Navier-Stokes equations. This is primarily due to the presence in the former in addition to nonlinear inertial terms of further strong nonlinearities owing to the complex dependence of pseudoturbulent stresses and other parameters on the intensity of pseudoturbulent pulsations and on dynamic variables and their derivatives. Such nonlinearities are, obviously, important only in cases of reasonably strong pseudoturbulence.

Simple estimates show that the pseudoturbulent terms can be neglected only in investigations of highly rarified disperse systems ($\langle\rho\rangle \leqslant 0.01 - 0.1$) and, also, of suspensions of very small particles (of a radius $\sim 10^{-4} - 10^{-3}$ cm) in a sufficiently viscous fluid. In all other cases the pseudoturbulent terms appearing in dynamic equations are comparable to or exceed the ordinary terms. However, as a matter of principle, even when the pseudoturbulent terms are small, they must be taken into consideration.

A complete disregard of the pseudoturbulent terms in dynamic equations would, in fact, result in the system of these equations to be, generally speaking, incomplete, thus necessitating the formulation of a certain additional relationship defining the equation of state of the disperse system and relating its concentration to other dynamic variables, if the quantities $\langle\rho\rangle$, $\langle p\rangle$, $\langle\mathbf{v}\rangle$ and $\langle\mathbf{w}\rangle$ are to be simultaneously determined. A good example is provided by the flow of suspension in a circular pipe considered in [4]. This complication is entirely naturally avoided by taking into consideration in dynamic equations the pseudoturbulent terms, no matter how small these are.

In solving specific problems it is necessary to take into account that the boundary conditions imposed on the solutions of Eqs. (2.18) differ considerably in the general case from those imposed on solutions of equations of hydrodynamics of monophase media. A detailed discussion of the form of boundary conditions is obviously outside the scope of this paper; only the boundary conditions which are to be specified at the solid walls bounding the flow are considered here. The normal velocity components $\langle\mathbf{v}\rangle$ and $\langle\mathbf{w}\rangle$ must obviously vanish at the solid boundaries. This statement does not, however, hold for the tangential components of $\langle\mathbf{v}\rangle$ and $\langle\mathbf{w}\rangle$. Solid particles are, in fact, capable of slip in the boundary neighborhood and of entraining adjacent volumes of fluid, which results in that the mean velocities of motion are not zero.

The no-slip condition of the thin interlayer of fluid separating the flow boundary from the disperse system is obviously satisfied but is not entirely mandatory for the fluid phase of such system at the interlayer boundary. This interlayer was introduced into the analysis in [4], where the conditions of phase velocities and stresses at its boundary with the disperse system were, also, considered.

An example of a problem involving a different kind of boundary conditions is provided by the problem of suspended material distribution and internal circulation of phases in a liquidized bed. The is bounded from below by a grid permeable only to the dispersing medium whose stream Q is normal to the grid. The first pair of boundary conditions is evidently of the form

$$\langle v_n \rangle = Q/1 - \langle \rho \rangle, \qquad \langle w_n \rangle = 0, \qquad x_n = 0 \tag{3.14}$$

It seems expedient to take as the second pair of boundary conditions those which define the pulsations of fluid penetrating through the grid, i.e.

$$\langle v_n'^2 \rangle = (\mathbf{R}[\mathbf{v}, \mathbf{w}]\,\theta)_{nn} = \vartheta Q^2, \quad \langle v_t'^2 \rangle = (\mathbf{R}\,[\mathbf{v}, \mathbf{w}]\,\theta)_{tt} = 0, \quad x_n = 0 \tag{3.15}$$

where ϑ is a certain coefficient dependent on the grid structure and the perturbations induced by it in the fluid flow. The quantities R and θ appearing in (3.15) in terms of dynamic variables and their derivatives obtained from relationships considered in the foregoing.

Examination of Eqs. (2.18) shows that it is possible to derive the expressions for subsequent terms of expansion (2.3). Each of these terms can be presented as the sum of the particular solution of the related nonhomogeneous equation (2.15) and of the general solution of the homogeneous equation, i.e.

$$f_m = f_m' + K_m \exp\left(-\frac{1}{2}\sum_{j=1}^{3}\frac{w_j'^2}{\theta_0^{(j)}}\right) \tag{3.16}$$

The first of the conditions (2.17) can, obviously, be always satisfied by a suitable selection of the constant K_m appearing in (3.16). However, the second of conditions (2.17) may not necessarily be identically satisfied from a certain number $m = M$ onwards. This has a profound physical meaning.

In fact, as shown in [5], there are generally no conditions whose fulfilment would make possible the simultaneous use of the set of equations of motion of particles and the statistical concept of "diffusion in the velocity field" used in the formulation of the kinetic equation for a system of particles. Hence it can be a priori expected that a unique corresondence between the definitions of suspended particles by kinetic equations and by momentum equations of the kind of (0.3) would be only within the accuracy of a certain approximation dependent on the deviation of the actual state of a disperse system from that of its equilibrium. The number M determines the properties of this approximation.

We note that the choice of the first term of expansion (2.3) in the form (1.3) is not unique. The sum of function (1.3) and certain other functions of an order $\geqslant\varepsilon$ can, also, be used for defining function f_0. In particular, function f_0 can be written in the form

$$f_0 = n\left(\frac{(\theta_0/\theta)^3}{8\pi^3\theta_0^{(1)}\theta_0^{(2)}\theta_0^{(3)}}\right)^{1/2}\exp\left(-\frac{1}{2}\frac{\theta_0}{\theta}\sum_{j=1}^{3}\frac{w_j'^2}{\theta_0^{(j)}}\right) \tag{3.17}$$

where θ is considered to be an independent dynamic variable, similarly to the temperature in the kinetic theory of gas. It will be readily seen that, if Eq. (2.20) is used in conjunction with Eqs. (0.3) and (0.4), the introduction of θ does not, in principle, complicate the solution of the kinetic equation (0.1). Although a system of successive hydromechanical approximations can be obtained by this method (it was used in [1] for deriving the Euler approximation), the latter is somewhat artificial, since, as shown in the foregoing, the pseudoturbulent energy of suspended particles, unlike the temperature of gas, cannot be considered as an independent parameter.

Kinetic equations can be similarly solved in the case in which the disperse system cannot be considered to be collision-free, however calculations become much more complicated. Consideration of collisions between particles is essential only in systems whose concentration does hardly differ from the close-packed state. The effect of collisions can be allowed for in the first approximation by introducing in the kinetic equation the usual Boltzmann term. An example of such equation was considered in [6].

The results presented here can be easily extended to polydisperse systems with discrete or continuous particle distribution with respect to their dimensions or density. The method developed in [1], according to which each component of the dispersed phase is considered as an independent phase, can be used for this purpose.

4. Appendix. Determination of the total force acting on a particle. The expression used above for the force acting on an individual particle differs somewhat from that used earlier (see, e.g. [1, 6]). In deriving this expression for the force F we proceed from the definition of the force acting on a single Stokes particle in an unbounded viscous fluid which can be written as

$$\mathbf{F} = m\mathbf{g} + \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4, \qquad \mathbf{F}_1 = -\sigma \frac{\partial p}{\partial \mathbf{r}}$$

$$\mathbf{F}_2 = d_0\sigma\beta\mathbf{u}, \quad \mathbf{F}_3 = \frac{1}{2} d_0\sigma \frac{d\mathbf{u}}{dt}, \quad \mathbf{u} = \mathbf{v} - \mathbf{w}, \quad \beta = \frac{9\nu_0}{2a^2} \tag{4.1}$$

$$\mathbf{F}_4 = d_0\sigma\gamma \int_{-\infty}^{t} \frac{d\mathbf{u}}{dt}\bigg|_{t=t'} \frac{dt'}{\sqrt{t-t'}}, \qquad \gamma = \frac{9}{2a}\left(\frac{\nu_0}{\pi}\right)^{1/2}$$

Here a is the particle radius, and the differentiation with respect to time is carried out along the particle trajectory. The velocity and pressure gradients in the fluid undisturbed by the presence of particles are denoted by $\mathbf{v}$ and ∇p, respectively. It is further assumed that the scale of variation of $\mathbf{v}$ and p considerably exceeds a.

The extension of expression (3.1) to cases in which, first, the Reynolds number which defines the flow around particles cannot be considered small and, second, when the fluid contains a great number of suspended particles, thus reducing the distance between these to dimensions comparable to their radius, makes it necessary to solve the problem of fluid flow through a grid of pulsating particles. Since so far, such solution has not been found, we shall limit our analysis to the phenomenological aspects of this problem.

It is, first of all, obvious that, independently of the presence of other particles in the (single-particle) system, the force $m\mathbf{g}$ excercised by the external field does not vary, while force $\mathbf{F}_1$ produced by the undisturbed pressure field in the fluid is, as before, expressed by

$$\mathbf{F}_1(t, r) = \oint p(\mathbf{r}' - \mathbf{r})\, dS = -\sigma \frac{\partial p}{\partial \mathbf{r}}, \qquad \sigma = \frac{4}{3}\pi a^3 \tag{4.2}$$

where integration is carried out over the particle surface. Thus the (expression for) force $\mathbf{F}_1$ remains of the same form even in the presence of other particles in the stream.

We define the stationary force of resistance to the motion of the single particle exerted by the fluid at high Reynolds numbers and nonzero $\langle\rho\rangle$, by the expression experimentally derived in hydraulics for a fluid flowing through a layer of immobilized particles. A survey of experimental data on F_1 is given in [7]. For the sake of definiteness we use here the Ergan relationship

$$\mathbf{F}_2 = d_0\sigma\,[\beta_1 K_1(\rho) + \beta_2 K_2(\rho)\,u]\,\mathbf{u}, \qquad \beta_1 = 9\nu_0/2a^2, \qquad \beta_2 = 0.165/a \tag{4.3}$$

Here β_1 and β_2 are coefficients dependent only on physical parameters of phases, such that (4.3) directly yields the expressions for the force under conditions of laminar or turbulent flow past the particle, when $K_1 = 1$ and $K_2 = 0$ or $K_1 = 0$ and $K_2 = 1$, respectively. Only the first term in (4.3) which defines resistance in the form given by Stokes remains in (4.3) when $R = 2au/\nu_0 \ll 1$, while for $R \gg 1$ only the second term defining the quadratic law of resistance is present. Functions $K_1(\rho)$ and $K_2(\rho)$ define the effect of constricted flow past an individual particle, resulting from the presence of neighboring particles. These functions must obviously become equal to unity when $\rho \to 0$. There are, also, empirical definitions of $K_1(\rho)$ and $K_2(\rho)$ [7]. Force F_3 related to the effect of acceleration of the apparent mass of fluid on the relative motion of a particle may, by analogy to (4.1), be represented in one of the following two forms:

$$\mathbf{F}_3 = d_0\sigma\xi(\rho)\frac{d\mathbf{u}}{dt} \qquad \text{or} \qquad \mathbf{F}_3 = d_0\sigma\frac{d}{dt}(\xi(\rho)\,\mathbf{u}) \tag{4.4}$$

where $\xi(\rho)$ – the substitute for the factor $1/2$ in (4.1) – is a certain function of local concentration in the disperse system. Unfortunately any experimental or theoretical data on ξ in a concentrated disperse system are not available.

To estimate approximately the factor $\xi(\rho)$ we shall consider the flow of a perfect fluid past a single particle of the grid, using the cellular model of constricted flow (see, e.g. [2]). Let us examine the relative motion of a particle at velocity u_* in a direction opposite to that of the x-axis. We use a system of spherical coordinates with its origin attached to the particle center at the considered instant of time, and establish the boundary conditions for the flow potential φ as

$$\left.\frac{\partial\varphi}{\partial r}\right|_{r=a} = -u_*\cos\theta, \qquad \left.\frac{\partial\varphi}{\partial r}\right|_{r=a'} = 0, \qquad a' = \frac{a}{\rho^{1/3}} \tag{4.5}$$

The solution of the Laplace equation for φ with conditions (4.5) is of the form

$$\varphi = \left(Ar + \frac{B}{r^2}\right)\cos\theta, \qquad A = \frac{\rho u_*}{1-\rho}, \qquad B = \frac{a^3 u_*}{2(1-\rho)}$$

$$\cos\theta = -(\mathbf{r}\mathbf{u}_*)/(ru_*) \tag{4.6}$$

The expression for the kinetic energy of the perturbed fluid flow inside a cell is written as

$$E = \frac{d_0}{2}\int\limits_{a<r<a'} |\nabla\varphi|^2\,dr = \frac{\sigma d_0 u_*^2}{4}\left[1 + \frac{\rho}{1-\rho}\left(3 + \frac{2/3}{1-\rho}\ln\rho\right)\right] \tag{4.7}$$

In the system of coordinates attached to a moving particle the fluid velocity is, obviously, equal to $u_* + \nabla\varphi$. The mean velocity of fluid in the cell can then be expressed in the form

$$u_x = \frac{1}{\sigma_\rho - \sigma}\int\limits_{a<r<a'}\left(u_* + \frac{\partial\varphi}{\partial x}\right)dr = u_*\left[1 + \frac{\rho}{1-\rho}\left(1 + \frac{1/3}{1-\rho}\ln\rho\right)\right] = u$$

$$u_y = u_z = 0, \qquad \{x, y, z\} = \{r_1, r_2, r_3\}, \qquad \sigma_\rho = \sigma/\rho \tag{4.8}$$

Expression (4.8) makes it possible to represent the value of (4.7) as a function of u. The total momentum of the perturbed flow can be readily derived from (4.7). As the result, after conventional calculations, we obtain for force F_3 the second relationships (4.4) in which

$$\xi(\rho)=\frac{1}{2}\left[1+\frac{\rho}{1-\rho}\left(3+\frac{2/3}{1-\rho}\right)\ln\rho\right]\left[1+\frac{\rho}{1-\rho}\left(1+\frac{1/3}{1-\rho}\ln\rho\right)\right]^{-1} \quad (4.9)$$

With ρ increasing from zero $\xi(\rho)$ first decreases from $1/2$, reaches its minimum and, then, begins to increase (reaching $7/4$ for $\rho\to 1$)

A phenomenologically reasonable generalization of the formula for F_4 in (4.1) can be written in the form

$$F_4=d_0\sigma\gamma\int_{t-t_0}^{t}\eta(\rho)\left.\frac{du}{dt}\right|_{t=t'}\frac{dt'}{\sqrt{t-t'}} \quad (4.10)$$

where $\eta(\rho)$ is a certain function of the order of unity, and t_0 is a characteristic time interval during which variation of local fluid velocity in the free volume around the particle are still affecting the force exerted on this particle. The value of t_0 can be considered to be the time of propagation of perturbation induced by the particle in the fluid flow between the particle and the cell surfaces. Since the basic concept of the lattice-like model is that perturbations outside the lattice do not affect the processes taking place inside it, this estimate of t_0 is entirely justified. The rate of wave propagation from an oscillating body in a viscous fluid is of the order of $(\nu_0\omega)^{1/2}$, where ω is the frequency of oscillation. In the case considered here ω is to be taken as the frequency of variation of u. Hence

$$t_0\sim\frac{a'-a}{(\nu_0\omega)^{1/2}}=\frac{1-\rho^{1/3}}{\rho^{1/3}}\,\frac{a}{(\nu_0\omega)^{1/2}} \quad (4.11)$$

As expected, the time $t_0\to\infty$, when $\rho\to 0$ in accordance with the expression (4.1) for an isolated particle. The interactions in a reasonably concentrated disperse system are of fundamental interest. We limit our considerations to systems for which $(1-\rho)^{1/3}\rho^{-1/3}\sim 1$–$10$.

Let us consider two limit cases. In the first of these

$$t_0\ll T\sim\frac{1}{\omega},\qquad \omega^{1/2}\ll\left(\frac{\nu_0}{a^2}\right)^{1/2} \quad (4.12)$$

which on the assumption that $u\sim a\omega$ becomes equivalent to inequality $R\ll 1$ which defines the inertia-free mode of flow past the particle. In this case, allowing for relationships (4.1) and (4.11), we obtain for the force (4.10) the estimate

$$F_4\sim d_0\sigma\gamma\omega u\sqrt{t_0}\sim d_0\sigma\left(\frac{\nu_0}{a}\right)^{1/4}\omega^{3/4}u\ll d_0\sigma\left(\frac{\nu_0}{a^2}\right)u\sim F_2 \quad (4.13)$$

hence this force can be neglected as small in comparison with the force of viscous interaction berween particle and fluid. In the second case the inequality (4.12) is reversed so that in (4.10) we can assume $t_0\approx\infty$. Substituting $u\cos\omega t$ for u and assuming $u\approx$ const, instead of (4.13) we obtain the estimate

$$F_4\sim d_0\sigma\gamma\sqrt{\omega}\,u\sim d_0\sigma\left(\frac{\nu_0\omega}{a^2}\right)^{1/2}u\ll d_0\sigma\omega u\sim F_3 \quad (4.14)$$

i.e. force F_4 can, again, be neglected as small in comparison with F_3. Estimates (4.13) and (4.14) that within a certain approximation force F_4 may be altogether excluded from the analysis. This exclusion is important, since taking this force into account would have meant a revision of the majority of relationships derived in the foregoing.

The kinetic equation (0.1) derived from the Kolmogorov-Chapman equation is, in fact, valid only on the assumption that the random variation of variables, which defines the local state of a disperse system, is a Markovian process, while, on the other hand, the dependence of force **F** on the history of particle motion is incompatible with that process. Although the $\mathbf{F}_4$ component of force **F** was taken into consideration in a number of papers (e.g. [1]), the limitations imposed by this on the admissibility of using the kinetic equation in the form (0.1) were ignored.

Relating force F to a unit of particle mass m, we obtain the equation for force **H**. Averaging the latter, we find the expression for $\langle \mathbf{H} \rangle$ appearing in (0.2) which is correct to terms of second order with respect to pseudoturbulent variables. Subtracting $\langle \mathbf{H} \rangle$ from **H**, we obtain the expression for the pulsating force $\mathbf{H}'$

$$\mathbf{H}' \approx \varkappa \Big\{ \beta_1 \left(K_1 \mathbf{u}' + \frac{dK_1}{d\langle \rho \rangle} \rho' \langle \mathbf{u} \rangle \right) + \beta_2 \left[K_2 (\langle u \rangle \mathbf{u}' + (\mathbf{u}_0 \mathbf{u}') \langle \mathbf{u} \rangle) + \frac{dK_2}{d\langle \rho \rangle} \rho' \langle u \rangle \langle \mathbf{u} \rangle \right] + $$
$$+ \frac{d}{dt} \left(\xi \mathbf{u}' + \frac{d\xi}{d\langle \rho \rangle} \rho' \langle \mathbf{u} \rangle \right) + \left(\mathbf{w}' \frac{\partial}{\partial \mathbf{r}} \right) \xi \langle \mathbf{u} \rangle \Big\} - \frac{1}{d_1} \frac{\partial p'}{\partial \mathbf{r}} \tag{4.15}$$

The expression for force $\mathbf{H}'$ contains terms proportional to various pseudoturbulent variables. Separating in expression (4.15) terms proportional to components $\mathbf{w}'$, we obtain for tensor c the relationship (0.2). The effect of remaining components of force $\mathbf{H}'$ on the random behavior of the dispersed phase particles is allowed for in conformity with the basic assumption used in the formulation of Eq. (0.1) by the introduction in it of the term which defines diffusion in a velocity space.

5. Appendix. The determination of properties of pseudoturbulence in the equilibrium approximation. The random pseudoturbulent variables satisfy the stochastic equations derived from the equation of motion of particles and fluid in the gap between particles [1]. The Langevin equation for a single particle is of the form

$$d\mathbf{w}'/dt = \mathbf{H}_0' \tag{5.1}$$

(the expression for $\mathbf{H}_0'$, valid for the equilibrium state, is derived from (4.15) by neglecting terms proportional to derivatives of dynamic variables).

The stochastic equations for the fluid are obtained by subtracting from the Navier-Stokes equation the related averaged equations [1]. For the equilibrium state this yields

$$\left(\frac{d}{dt} + \langle \mathbf{u} \rangle \frac{\partial}{\partial \mathbf{r}} \right) \rho' - (1 - \langle \rho \rangle) \frac{\partial \mathbf{v}'}{\partial \mathbf{r}} = 0 \tag{5.2}$$
$$\left(\frac{d}{dt} + \langle \mathbf{u} \rangle \frac{\partial}{\partial \mathbf{r}} \right) \mathbf{v}' = - \frac{1}{d_0 (1 - \langle \rho \rangle)} \frac{\partial p'}{\partial \mathbf{r}} + \frac{2 \nu_0 S}{1 - \langle \rho \rangle} \frac{\partial \mathbf{e}_i'}{\partial \mathbf{r}} - \frac{\langle \rho \rangle}{\varkappa (1 - \langle \rho \rangle)} \mathbf{H}'$$

Changes of parameters p', $\mathbf{v}'$ and $\mathbf{w}'$ can be estimated from Eqs. (5.1) and (5.2) for a given random function $\rho'(t, \mathbf{r})$ which defines fluctuations of local concentration in the disperse system, i.e. it becomes possible to express the statistical properties of these parameters in terms of related characteristics of ρ'. The simplest way of achieving this is to use the correlation theory of stationary random processes, which implies presenting ρ', p', $\mathbf{v}'$ and $\mathbf{w}'$ in the form of Fourier-Stieltjes integrals and examine the spectral measures appearing in these. From (5.2) we derive for the spectral measures the following system of equations: $i\omega d\mathbf{Z}_w = d\mathbf{Z}_H$, $\quad (\omega + \langle \mathbf{u} \rangle \mathbf{k}) dZ_\rho - (1 - \langle \rho \rangle) \mathbf{k} d\mathbf{Z}_v = 0 \quad (5.3)$

$$(\omega - \langle \mathbf{u} \rangle \mathbf{k})\, d\mathbf{Z}_v = - \frac{i\mathbf{k}}{d_1 (1 - \langle \rho \rangle)} dZ_p - \frac{\nu_0 S}{1 - \langle \rho \rangle} \left[k^2 d\mathbf{Z}_v + \frac{1}{3} \mathbf{k} (\mathbf{k} d\mathbf{Z}_v) \right] - \frac{\langle \rho \rangle}{\varkappa (1 - \langle \rho \rangle)} d\mathbf{Z}_H$$

From (4.15) for dZ_H we have

$$dZ_H = \varkappa \Big\{ (\beta_1 K_1 + \beta_2 K_2 \langle u \rangle) \, dZ_u + \beta_2 K_3 (\mathbf{u}_0 dZ_u) \langle \mathbf{u} \rangle + \\ + \Big(\beta_1 \frac{dK_1}{d\langle \rho \rangle} + \beta_2 \frac{dK_2}{d\langle \rho \rangle} \langle u \rangle \Big) \langle \mathbf{u} \rangle \, dZ_\rho + i\omega\xi dZ_u + \\ + i\omega \frac{d\xi}{d\langle \rho \rangle} \langle \mathbf{u} \rangle \, dZ_\rho \Big\} - \frac{i\mathbf{k}}{d_1} dZ_p, \qquad dZ_u = dZ_v - dZ_w \tag{5.4}$$

These equations make it possible to express all spectral measures and, also, all spectral densities in terms of spectral measure dZ_ρ and of spectral density $\Psi_{\rho,\rho}(\omega, \mathbf{k})$ of the random process ρ'. The space-time correlation functions of the various pseudoturbulent processes are derived from the related spectral densities by integrating the latter with respect to frequencies and the pulsation wave vector $\mathbf{k}$. Integration is carried out in the conventional manner, and the necessary definition of $\Psi_{\rho,\rho}(\omega, \mathbf{k})$ can, for example, be taken from [8]. As the result, we obtain in the equilibrium approximation all characteristics of pseudoturbulence, which appear in dynamic equations considered above, expressed in the form of explicit functions of dynanic variables.

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